TAM 511 Homework set ① 1/21/09

1. Let $\{\underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}\}$ denote the usual orthonormal basis for $\mathbb{R}^3$, viz., $\underset{\sim}{i} = (1,0,0)$, $\underset{\sim}{j} = (0,1,0)$, $\underset{\sim}{k} = (0,0,1)$. Define

$$\underset{\sim}{g}_1 \equiv 2\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}, \quad \underset{\sim}{g}_2 \equiv -\underset{\sim}{i} + \underset{\sim}{k}, \quad \underset{\sim}{g}_3 \equiv 4\underset{\sim}{i} - \underset{\sim}{j} + 6\underset{\sim}{k}.$$

(a) Show that $\{\underset{\sim}{g}_1, \underset{\sim}{g}_2, \underset{\sim}{g}_3\}$ is a basis for $\mathbb{R}^3$.

(b) Find the covariant and contravariant components of $\underset{\sim}{v} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$ relative to $\{\underset{\sim}{g}_1, \underset{\sim}{g}_2, \underset{\sim}{g}_3\}$ and $\{\underset{\sim}{g}^1, \underset{\sim}{g}^2, \underset{\sim}{g}^3\}$, respectively.

2. Let $\mathcal{P}^2(0,1)$ denote the set of all real-valued polynomials of degree 2 or less, restricted to the unit interval, i.e.,

$$\mathcal{P}^2(0,1) \equiv \{a_0 + a_1 x + a_2 x^2 : a_0, a_1, a_2 \in \mathbb{R}, \; 0 < x < 1\}.$$

Define $$\underset{\sim}{p}_1 \cdot \underset{\sim}{p}_2 \equiv \int_0^1 \underset{\sim}{p}_1(x)\, \underset{\sim}{p}_2(x)\, dx,$$

$\forall\, \underset{\sim}{p}_1, \underset{\sim}{p}_2 \in \mathcal{P}^2(0,1)$, and

$$\underset{\sim}{g}_1(x) \equiv 1, \quad \underset{\sim}{g}_2(x) \equiv x, \quad \underset{\sim}{g}_3(x) \equiv x^2.$$

(a) Show that $\{\underset{\sim}{g}_1, \underset{\sim}{g}_2, \underset{\sim}{g}_3\}$ is a basis for $\mathcal{P}^2(0,1)$.

(b) Determine $\underset{\sim}{g}^1(x)$, $\underset{\sim}{g}^2(x)$, $\underset{\sim}{g}^3(x)$.