

- ① Show that any solution of the E-L equation for

$$I(y) = \int_{-1}^1 x \sqrt{1 + (y')^2} dx$$

$$y(-1) = \alpha, \quad y(1) = \beta,$$

is not a minimizer.

- ② Consider again the column buckling problem (p. 173) under tension  $T \geq 0$ .

$$I(y) = \int_0^1 \left[ \frac{1}{2} (y')^2 - T \cos y \right] dx$$

$$y(0) = y(1) = 0$$

Check to see if the trivial solution is a local minimizer.

- ③ Consider the basic functional (p. 160), in which  $y$  is absent from the integrand:

$$I(y) = \int_a^b f(x, y') dx,$$

$$y(a) = y_a, \quad y(b) = y_b.$$

Assuming Legendre's strengthened condition, show there are no conjugate points.