TAM 611 Homework Set (11) 4/10/09

(1) Consider the elastic beam potential (p. 112)

$$I(y) = \int_0^1 \left[\frac{1}{2}(y'')^2 - qy\right] dx$$

$$y(0) = y'(0) = 0$$

(a) Use the Rayleigh-Ritz method with

$$y = a\,\phi_1(x) = a\left[1 - \cos\left(\frac{\pi x}{2}\right)\right]$$

and determine "a" by energy minimization.

(b) Compare $y(1) = a\,\phi_1(1)$ with the exact solution $y(1)$ (cf. p. 114).

(2) Consider the plate functional for a simply-supported square plate, $\Omega = (0,1) \times (0,1)$ subject to a constant load $\hat{p}$ (cf. p 139):

$$I(w) = \iint_\Omega \frac{1}{2} D(\Delta w)^2\, dx\,dy - \hat{p} \iint_\Omega w\, dx\,dy,$$

$$w\big|_{\partial\Omega} = 0.$$

(a) Use the Rayleigh Ritz method with

$$w = a_{11} \sin(\pi x) \sin(\pi y)$$ to determine "a_{11}".

(b) Repeat (a) for

$$w = \sum_{n=1}^{N} \sum_{m=1}^{N} a_{nm} \sin(n\pi x) \sin(m\pi y)$$ (over)

to find the coefficients q_{nm}, $n, m = 1, 2, \ldots, N$ for any positive integer N.

(**Hint:** $\displaystyle\int_0^1 \sin(p\tau)\sin(q\tau)\,d\tau = 0 \quad p \neq q$)