TAM 611 Homework Set (13) 4/22/09

(1) Consider the "finite-rod" problem

$$u_{tt} = c^2 u_{xx} \qquad 0<x<l$$
$$u(0,t) = u(l,t) = 0 \qquad t>0$$
$$\hat{\phi}(x) = 0$$
$$\hat{\psi}(x) \neq 0.$$

Solve the problem via D'Alembert's solution (travelling waves) and show that this is the same as the separation-of-variables or Fourier solution (cf. pp. 219–222).

(2) Suppose that $u(x,y)$ is harmonic in the disk $D = \{(r,\theta) : r<2,\ 0\le\theta<2\pi\}$ with $u(2,\theta) = 1+3\sin 2\theta$.

(a) Find the maximum value of u in $\overline{D}$.

(b) Find the value of u at the center $(x,y)=(0,0)$.

(3) Solve $\Delta u = 0$ in the half-disk

$$\Omega = \{(r,\theta) : r<1,\ 0<\theta<\pi\},$$

subject to $u(r,0) = u(r,\pi) = 0$

$u(1,\theta) = \pi\sin\theta - \sin 2\theta.$