TAM 611 Homework Set 14 4/29/09

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1) solve

$$u_t = \alpha^2 u_{xx} \qquad -\infty < x < \infty$$

$$t > 0$$

$$u(x,0) = \begin{cases} 1 & x > 0 \\ 3 & x < 0 \end{cases}$$

Express your answer in terms of $\mathrm{erf}(\cdot)$.

2) (a) Consider the semi-infinite problem

$$\left.\begin{aligned} u_t &= \alpha^2 u_{xx} \\ u_x(0,t) &= 0 \\ u(x,0) &= \hat{\phi}(x) \end{aligned}\right\} \quad \begin{aligned} 0 < x < \infty \\ t > 0 \end{aligned}$$

(The b.c. represents a perfectly insulated end – zero "heat flux" there.)

Take the appropriate (even) extension and obtain the general form of the solution (along the lines of (*) p. 256).

(b) Show that the solution is even (hence satisfies the b.c.)

(c) Solve for the case $\hat{\phi}(x) = 1$

3) Use the energy method to prove uniqueness for the Neumann IBVP:

$$\left.\begin{array}{l} u_t = \alpha^2 u_{xx} + \hat{f}(x,t) \\ u_x(0,t) = \hat{g}(t) \\ u_x(\ell,t) = \hat{h}(t) \\ u(x,0) = \hat{\phi}(x). \end{array}\right\} \begin{array}{l} 0 < x < \ell \\ t > 0. \end{array}$$

(4) Consider the functional

$$I(y) = \int_0^1 (y')^2 dx \quad , \text{ subject to}$$

$$\int_0^1 y^2 dx = 1.$$

(a) Use the Lagrange mult. method to obtain the E-L (eigenvalue) problem. (Hint: don't forget the natural b.c.'s!)

(b) solve the 2-point boundary eigenvalue problem.

(5) Use separation of variables to solve the IBVP

$$\left.\begin{array}{l} u_t = \alpha^2 u_{xx} \\ u_x(0,t) = u_x(\ell,t) = 0 \\ u(x,t) = 8\cos^2 x \end{array}\right\} \begin{array}{l} 0 < x < \ell \\ t > 0 \end{array}$$

(Hint: This is just as easy as Prob (3) of HW (13).)