Homework ② solutions

① $\underset{\sim}{x} = x^i \underset{\sim}{g}_i$

$$\underset{\sim}{A}(x^i \underset{\sim}{g}_i) = x^i \underset{\sim}{A}\underset{\sim}{g}_i = x^1(5\underset{\sim}{g}^2) + x^2(\underset{\sim}{g}^1 - \underset{\sim}{g}^3) + x^3(\underset{\sim}{g}^1 + \underset{\sim}{g}^2 + \underset{\sim}{g}^3)$$

$$\Rightarrow \underset{\sim}{A}\underset{\sim}{x} = (x^2 + x^3)\underset{\sim}{g}^1 + (5x^1 + x^3)\underset{\sim}{g}^2 + (-x^2 + x^3)\underset{\sim}{g}^3$$

$\Rightarrow$ ① $x^2 + x^3 = 5$

② $5x^1 + x^3 = -3$

③ $-x^2 + x^3 = 1$

① + ② $\Rightarrow$ $2x^3 = 6 \Rightarrow \underline{x^3 = 3}$

① $\Rightarrow \underline{x^2 = 5 - x^3 = 2}$

② $\Rightarrow x^1 = \frac{-3 - x^3}{5} = -\frac{6}{5}$

$\therefore$ $\boxed{\underset{\sim}{x} = -\frac{6}{5}\underset{\sim}{g}_1 + 2\underset{\sim}{g}_2 + 3\underset{\sim}{g}_3}$

② Consider $\underset{\sim}{T} \cdot \underset{\sim}{S} \equiv \not{tr}(\underset{\sim}{T}\underset{\sim}{S}^T)$

Prop(i) p.27 $= \not{tr}((\underset{\sim}{T}\underset{\sim}{S}^T)^T)$

Prop. (i),(ii) p.25 $= \not{tr}(\underset{\sim}{S}\underset{\sim}{T}^T)$. □

③ Claim ② p. 28 $\Rightarrow$

$$(\underset{\sim}{g}_i \otimes \underset{\sim}{g}_j)\cdot(\underset{\sim}{g}^k \otimes \underset{\sim}{g}^l) = (\underset{\sim}{g}_i \cdot \underset{\sim}{g}^k)(\underset{\sim}{g}_j \cdot \underset{\sim}{g}^l) = \delta_i^k \delta_j^l$$

Similarly $(\underset{\sim}{g}_i \otimes \underset{\sim}{g}^j)\cdot(\underset{\sim}{g}^k \otimes \underset{\sim}{g}_l) = \delta_i^k \delta_l^j$.

Thus, $\underset{\sim}{g}_i \otimes \underset{\sim}{g}_j \cdot \underset{\sim}{T}$

$$= \underset{\sim}{g}_i \otimes \underset{\sim}{g}_j \cdot \left(T_{kl} \underset{\sim}{g}^k \otimes \underset{\sim}{g}^l \right)$$

$$= T_{kl} \delta_i^k \delta_j^l = T_{ij},$$

and

$$\underset{\sim}{g}_i \otimes \underset{\sim}{g}^j \cdot \underset{\sim}{T}$$

$$= \underset{\sim}{g}_i \otimes \underset{\sim}{g}^j \cdot \left(T_k{}^l \underset{\sim}{g}^k \otimes \underset{\sim}{g}_l \right)$$

$$= T_k{}^l \delta_i^k \delta_l^j = T_i{}^j.$$