TAM 611 Homework set (2) 1/28/09

(1) Given $\{\underset{\sim}{g}_1, \underset{\sim}{g}_2, \underset{\sim}{g}_3\}$, $\{\underset{\sim}{g}^1, \underset{\sim}{g}^2, \underset{\sim}{g}^3\}$, a biorthonormal set of bases for a 3-dimensional vector space $\mathcal{V}^3$ (real, inner-product space), suppose $\underset{\sim}{A} \in \mathcal{L}(\mathcal{V}^3)$ s.t.,

$$\underset{\sim}{A}\underset{\sim}{g}_1 = 5\underset{\sim}{g}^2, \quad \underset{\sim}{A}\underset{\sim}{g}_2 = \underset{\sim}{g}^1 - \underset{\sim}{g}^3$$

$$\underset{\sim}{A}\underset{\sim}{g}_3 = \underset{\sim}{g}^1 + \underset{\sim}{g}^2 + \underset{\sim}{g}^3 .$$

Solve for $\underset{\sim}{x}$ if $\underset{\sim}{A}\underset{\sim}{x} = 5\underset{\sim}{g}^1 - 3\underset{\sim}{g}^2 + \underset{\sim}{g}^3$.

(2) Prove "Claim (1)" on p. 28.

(3) From class we know that $\{\underset{\sim}{g}_i \otimes \underset{\sim}{g}_j\}$, $\{\underset{\sim}{g}^i \otimes \underset{\sim}{g}^j\}$, $\{\underset{\sim}{g}_i \otimes \underset{\sim}{g}^j\}$ and $\{\underset{\sim}{g}^i \otimes \underset{\sim}{g}_j\}$ each form a basis for $\mathcal{L}(\mathcal{V})$, cf. pp. 19-20 of the notes.

Show that $\{\underset{\sim}{g}_i \otimes \underset{\sim}{g}_j\}$ and $\{\underset{\sim}{g}^i \otimes \underset{\sim}{g}^j\}$ are a biorthonormal pair of bases for $\mathcal{L}(\mathcal{V})$. Likewise show that $\{\underset{\sim}{g}_i \otimes \underset{\sim}{g}^j\}$ and $\{\underset{\sim}{g}^i \otimes \underset{\sim}{g}_j\}$ are biorthonormal, viz.,

$$(\underset{\sim}{g}_i \otimes \underset{\sim}{g}_j) \cdot (\underset{\sim}{g}^k \otimes \underset{\sim}{g}^l) = \delta_i^k \delta_j^l$$

and

$$(\underset{\sim}{g}_i \otimes \underset{\sim}{g}^j) \cdot (\underset{\sim}{g}^k \otimes \underset{\sim}{g}_l) = \delta_i^k \delta_l^j .$$

Use this to show, e.g., that $T_{ij} = (\underset{\sim}{g}_i \otimes \underset{\sim}{g}_j) \cdot \underset{\sim}{T}$ and $T_i{}^j = (\underset{\sim}{g}_i \otimes \underset{\sim}{g}^j) \cdot \underset{\sim}{T}$.