TAM 6110 Homework Set (3) 2/4/09 1/2

Sp. 09 Healey

(1) Given $\underset{\sim}{a} = a^i g_i$ and $\underset{\sim}{b} = b^j \underset{\sim}{g}_j$, use the results of problem (3) of Homework set (2) to show that

$$(\underset{\sim}{a} \otimes \underset{\sim}{b}) \cdot \underset{\sim}{T} = \underset{\sim}{a} \cdot (\underset{\sim}{T}\underset{\sim}{b}) \quad \forall \underset{\sim}{T} \in L(\mathcal{V}), \; \underset{\sim}{a}, \underset{\sim}{b} \in \mathcal{V}.$$

(2) An alternative approach to 4th-order tensors (compared to our approach in class, cf. pp. 34-36) is the following:

Defn. 1. A 4th-order tensor is a mapping $\overset{(4)}{\underset{\sim}{T}} : L(\mathcal{V}) \to L(\mathcal{V})$ that is linear, i.e.,

$$\overset{(4)}{\underset{\sim}{T}}(\alpha \underset{\sim}{A} + \beta \underset{\sim}{B}) = \alpha \overset{(4)}{\underset{\sim}{T}}(\underset{\sim}{A}) + \beta \overset{(4)}{\underset{\sim}{T}}(\underset{\sim}{B}) \quad \forall \underset{\sim}{A}, \underset{\sim}{B} \in L(\mathcal{V}), \; \alpha, \beta \in \mathbb{R}.$$

Defn. 2. The tensor product of two dyadic products $\underset{\sim}{a} \otimes \underset{\sim}{b}$ and $\underset{\sim}{c} \otimes \underset{\sim}{d}$ is defined by

$$(\underset{\sim}{a} \otimes \underset{\sim}{b} \otimes \underset{\sim}{c} \otimes \underset{\sim}{d})\underset{\sim}{A} \equiv \underset{\sim}{a} \otimes \underset{\sim}{b}\left((\underset{\sim}{c} \otimes \underset{\sim}{d}) \cdot \underset{\sim}{A}\right) \quad \forall \underset{\sim}{A} \in L(\mathcal{V}).$$

(a) Based upon these definitions, show that, e.g.,

$$\overset{(4)}{\underset{\sim}{T}} = T^{ijkl} \underset{\sim}{g}_i \otimes \underset{\sim}{g}_j \otimes \underset{\sim}{g}_k \otimes \underset{\sim}{g}_l,$$

where $T^{ijkl} = (\underset{\sim}{g}^i \otimes \underset{\sim}{g}^j) \cdot \underset{\sim}{\overset{(4)}{T}} (\underset{\sim}{g}^k \otimes \underset{\sim}{g}^l)$, for every $\underset{\sim}{\overset{(4)}{T}} \in L(L(\mathcal{V}), L(\mathcal{V})) = \overset{(4)}{L}(\mathcal{V})$.

(b) Show that, e.g.,

$$\underset{\sim}{\overset{(4)}{T}} \underset{\sim}{A} = T^{ijkl} A_{kl} \, \underset{\sim}{g}_i \otimes \underset{\sim}{g}_j \quad \forall \underset{\sim}{\overset{(4)}{T}} \in \overset{(4)}{L}(\mathcal{V}), \; \underset{\sim}{A} \in L(\mathcal{V}).$$

③ If $f(\underset{\sim}{r}) \equiv \exp(|\underset{\sim}{r}|)$, compute

$\underset{\sim}{\nabla} f(\underset{\sim}{r})$ (no coordinates allowed!)

$(\underset{\sim}{r} \neq \underset{\sim}{0})$