

# Homework (3) Solutions

1/2

$$\textcircled{1} \quad \underline{g}_i \otimes \underline{g}_j \cdot \underline{T} = \underset{\substack{\uparrow \\ \text{Homework (2)}}}{T_{ij}} = \underset{\substack{\uparrow \\ \text{p. 18 Lect. Notes}}}{\underline{g}_i \cdot \underline{T} \underline{g}_j}$$

$$\begin{aligned} \therefore (\underline{a} \otimes \underline{b}) \cdot \underline{T} &= a^i b^j (\underline{g}_i \otimes \underline{g}_j) \cdot \underline{T} \\ &= a^i b^j \underline{g}_i \cdot \underline{T} \underline{g}_j \\ &= a^i \underline{g}_i \cdot \underline{T} (b^j \underline{g}_j) = \underline{a} \cdot \underline{T} \underline{b}. \quad \square \end{aligned}$$

$$\begin{aligned} \textcircled{2} \text{ (a)} \quad \underline{T}^{(4)}(\underline{A}) &= \underline{T}^{(4)}(A_{kl} \underline{g}^k \otimes \underline{g}^l) \stackrel{\text{Defn. 1}}{=} A_{kl} \underline{T}^{(4)}(\underline{g}^k \otimes \underline{g}^l) \\ &= (\underline{g}^k \otimes \underline{g}^l) \cdot \underline{A} \underline{T}^{(4)}(\underline{g}^k \otimes \underline{g}^l) \\ &= (\underline{g}^i \otimes \underline{g}^j) \cdot \underline{T}^{(4)}(\underline{g}^k \otimes \underline{g}^l) \underline{g}_i \otimes \underline{g}_j (\underline{g}^k \otimes \underline{g}^l) \cdot \underline{A} \\ &\stackrel{\text{Defn. 2}}{=} T^{ijkl} (\underline{g}_i \otimes \underline{g}_j \otimes \underline{g}^k \otimes \underline{g}^l) \underline{A} \quad \forall \underline{A} \in \mathbb{R}^3 \end{aligned}$$

$$\Rightarrow \underline{T}^{(4)} = T^{ijkl} (\underline{g}_i \otimes \underline{g}_j \otimes \underline{g}^k \otimes \underline{g}^l)$$

$$\text{where } T^{ijkl} = (\underline{g}^i \otimes \underline{g}^j) \cdot \underline{T}^{(4)}(\underline{g}^k \otimes \underline{g}^l).$$

$$\begin{aligned} \text{(b)} \quad \underline{T}^{(4)} \underline{A} &= T^{ijkl} \underline{g}_i \otimes \underline{g}_j \otimes \underline{g}^k \otimes \underline{g}^l A_{mn} \underline{g}^m \otimes \underline{g}^n \\ &= T^{ijkl} A_{mn} \underline{g}_i \otimes \underline{g}_j (\underline{g}^k \otimes \underline{g}^l) \cdot (\underline{g}^m \otimes \underline{g}^n) \\ &= T^{ijkl} A_{mn} \underline{g}_i \otimes \underline{g}_j \delta_k^m \delta_l^n \\ &= T^{ijkl} A_{kl} \underline{g}_i \otimes \underline{g}_j. \quad \square \end{aligned}$$

$$\textcircled{3} \frac{d}{d\alpha} f(\underline{r} + \alpha \underline{n}) \Big|_{\alpha=0}$$

$$= \frac{d}{d\alpha} \exp\left(\left(\underline{r} + \alpha \underline{n}\right) \cdot \left(\underline{r} + \alpha \underline{n}\right)\right)^{1/2}$$

$$= \exp(|\underline{r}|) \frac{1}{\cancel{r} \cdot \underline{n}} \frac{\cancel{r} \cdot \underline{n}}{(\underline{r} \cdot \underline{r})^{1/2}}$$

$$= \exp(|\underline{r}|) \frac{\underline{r} \cdot \underline{n}}{|\underline{r}|} \quad \forall \underline{n}$$

$$\Rightarrow \boxed{\nabla f(\underline{r}) = \exp(|\underline{r}|) \frac{\underline{r}}{|\underline{r}|}}$$