

Homework Set ⑤

① Express Δu for $u(\rho, \phi, \theta) = u(x^1, x^2, x^3)$, where (ρ, ϕ, θ) are the spherical coordinates shown on p. 47 of the class notes.

② Consider a torus:

$$\underline{r} = (a + b \cos(x^1)) \cos(x^2) \underline{e}_1 + (a + b \cos(x^1)) \sin(x^2) \underline{e}_2 + b \sin(x^1) \underline{e}_3,$$

for $0 \leq x^1 < 2\pi$, $0 \leq x^2 < 2\pi$, where $0 < b < a$ are constants.

Show that the total surface area is $4\pi^2 ab$.

③ Consider the hyperbolic paraboloid $z = y^2 - x^2$, where (x, y, z) are the usual rectangular Cartesian coordinates.

(a) Determine $\underline{n}(\underline{r})$

(b) Verify that $\underline{\nabla} \underline{n}(\underline{0}) \underline{i} = 2 \underline{i}$ and $\underline{\nabla} \underline{n}(\underline{0}) \underline{j} = -2 \underline{j}$.

(c) Determine the principal curvatures k_1 & k_2 at $\underline{r} = \underline{0}$, and the corresponding principal directions.