TAM 611 Homework Set 6 2/26/09

1. Consider the Monge parametrization of a surface:

$$\underset{\sim}{r} = x\underline{e}_1 + y\underline{e}_2 + h(x,y)\,\underline{e}_3,$$

where $h : \mathcal{D} \subset \mathbb{R}^2 \to \mathbb{R}$ is smooth.

Show that
$$K = \frac{h_{xx}h_{yy} - (h_{xy})^2}{\left(1+(h_x)^2+(h_y)^2\right)^2} \quad \left(\begin{matrix}\text{Gaussian}\\ \text{curv.}\end{matrix}\right)$$

and
$$H = \frac{[1+(h_x)^2]h_{yy} - 2h_x h_y h_{xy} + [1+(h_y)^2]h_{xx}}{2\left[1+(h_x)^2+(h_y)^2\right]^{3/2}} \quad \left(\begin{matrix}\text{mean}\\ \text{curv.}\end{matrix}\right)$$

2. Consider all curves on a cylinder of radius "a" of the form

$$\underset{\sim}{r} = a\left[\cos(\theta)\underline{e}_1 + \sin(\theta)\underline{e}_2\right] + z(\theta)\,\underline{e}_3,$$

where $z(\theta)$ is $C^1$. (a) Among all such curves connecting the points $(\theta_1, z_1)$ and $(\theta_2, z_2)$ find the one that renders the total arc-length functional stationary (1st variation = 0). (b) What type of curve is this?

3. (a) Find the E-L equation for $I(y) = \int_0^1 [(y')^2 + (y')^3]\,dx$.
(b) Find a solution satisfying $y(0) = y(1) = 0$.