TAM 611 Homework Set (7) 3/4/09

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(1) (a) Find the E-L equation and the natural boundary conditions for the functional

$$I(y) = \frac{1}{2}\int_0^1 (y'')^2 dx - P y(1),$$
$$y(0) = y'(1) = 0,$$

where P is a prescribed constant (load). Here $I(y)$ represents the potential energy of the following beam problem:

(b) Find $y(x)$.

(2) Consider the Dido problem

$$\max\ I(y) = \int_{-1}^{1} y\, dx,$$
$$y(-1) = y(1) = 0,$$

Subject to $\int_{-1}^{1} \sqrt{1+(y')^2}\, dx = L,$

where $2 < L \le \pi$.

(a) Show that $y(x)$ is an arc of a circle.
(b) What is $y(x)$ when $L = \pi$?

(3) (a) Employing the coordinates on p. 72 for the unit sphere, show that the arc-length functional for curves on the sphere is

$$I(Y_1, Y_2) = \int_{t_0}^{t_1} \left[(Y_1')^2 + (Y_2')^2 \sin^2(Y_1) \right]^{1/2} dt$$

where $\phi = Y_1(t)$ and $\theta = Y_2(t)$ represents a curve on the sphere.

(b) Obtain the E-L equations for the functional in (a).

(c) Solve the E-L equations for the 2 special cases:

(i) $\theta = Y_2(t) \equiv \text{Const}$

(ii) $\phi = Y_1(t) \equiv \pi/2$