Homework ⑨ Solutions

① Assuming E-L is enforced, and using $\left.\frac{\partial \eta}{\partial x_1}\right|_{x_1=a} = 0$,

$$\delta I \overset{p.151}{=} D \int_0^b \left[-\frac{\partial}{\partial x_1}\left(\frac{\partial^2 w}{\partial x_1^2} + \frac{\partial^2 w}{\partial x_2^2}\right) - (1-\nu)\frac{\partial^3 w}{\partial x_1 \partial x_2^2}\right] \eta \Big|_{x_1=a} dx_2$$

$$- \int_0^b q\, \eta \Big|_{x_1=a} dx_2 = 0 \quad \forall \eta \in \mathcal{X}_0.$$

$$\Rightarrow \quad \left.\left[\frac{\partial^3 w}{\partial x_1^3} + 2(1-\nu)\frac{\partial^3 w}{\partial x_1 \partial x_2^2}\right]\right|_{x_1=a} = -\frac{q}{D} \quad 0 \le x_2 \le b.$$

② $$\delta I \overset{p.158,159}{=} \int_\Omega \left[-\underline{\nabla} \cdot \left(\underline{\nabla}_A W(\underline{\nabla}\underline{u})\right) + \underline{b}\right] \cdot \underset{\sim}{\eta}\, d\underset{\sim}{x}$$

$$+ \int_{\partial\Omega_2} \nabla_A W(\underline{\nabla}\underline{u})\, \underset{\sim}{n} \cdot \underset{\sim}{\eta}\, dS = 0 \quad \forall \underline{\eta} \in \mathcal{X}_0.$$

0 on $\partial\Omega_1$

Assuming E-L, and since $\underset{\sim}{\eta}\big|_{\partial\Omega_2}$ is arbitrary, we have

$$\underset{\sim}{\nabla}_A W(\underline{\nabla}\underline{u})\, \underset{\sim}{n}\Big|_{\partial\Omega_2} = \underset{\sim}{0}$$

or

$$\frac{\partial W}{\partial A_{ij}}(\underline{\nabla}\underline{u})\, n_j \Big|_{\partial\Omega_2} = 0 \quad i = 1, 2, 3.$$